Round or flat?

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1 Introduction

Have you ever questioned the roundness of Earth? Many of us accept it as a fact from our school days and do not dwell on it. While teachers could present additional evidence or discuss the implications of a flat Earth, such discussions might be time-consuming and not particularly beneficial.

It is our responsibility to provide you with some methods for measuring the roundness of the Earth using everyday items. It is important to note that our aim is not to definitively prove the roundness of the Earth, but rather to demonstrate that it is not implausible for the Earth to be round. With this in mind, we will present a series of basic experiments in the second Chapter. In Chapter 3, we will introduce an experiment that we have conducted, along with a discussion of the implications of our results. Additionally, we will present the relevant data in the appendix.

2 Basic experiments

2.1 Sea-level experiment

One prominent experiment featured a flat Earth theorist attempting to confirm the Earth's curvature using optical apparatus [www.youtube.com/watch?v=TmnZe34Xix8]. Their method was based on the concept of sea level, as shown in Figure (1). If the Earth were flat, it would theoretically be feasible to project a powerful laser beam along a coastline. If correctly aligned, the beam should maintain the same height over a significant distance. Mathematically in the Figure 1, the elevations H_1 and H_2 should remain unchanged regardless of the distance L. On the curved shape of the Earth, if the flashlight is placed parallel to the ground, it will shine above the detector.

This approach provides a user-friendly way to illustrate the Earth's curvature, although it necessitates a coastline spanning several kilometers and specialized tools such as a laser and altimeter. Furthermore, correctly setting it up can be challenging for non-professionals. Nonetheless, if you have companions who share an interest in science, this could be a fun activity to engage in together. Below, we will outline some fundamental calculations for accurately replicating the experiment showcasing the curvature of the Earth.

Let's start by looking at the equation of a circle with radius R:

$$x^2 + y^2 = R^2, \quad (1)$$

where x and y denote Cartesian coordinates. Due to the circular symmetry, the selection of initial coordinates is arbitrary. For simplicity, we set $y_0 = R$ and $x_0 = 0$ to represent the position of our detector. Subsequently, we determine the distance of the flashlight, denoted as L. A reasonable choice for this length is $L = 5$ km, which will represent the value of x in equation (1). With a known radius $R = 6378$ km, we can express y as

$$y = \sqrt{R^2 - x^2} = \sqrt{R^2 - L^2}.$$

We are in the final stages of our work. Our next step involves determining the difference between y_0 and y to determine the height of our flashlight relative to the detector.

$$\Delta y = y_0 - y = R - \sqrt{R^2 - L^2} = 1.96 \times 10^{-3}\, km = 1.96\, m \approx 2\, m \qquad (1)$$

Based on our calculations, the detector should be positioned approximately 2 meters higher than the detector. It is essential for the flashlight to be positioned completely horizontally for proper functionality. To ensure this, we can incorporate two diaphragms, illustrated in figure 1, to prevent the flashlight from illuminating the detector at an angle other than completely horizontal.

From a mathematical point of view, we should point out, that L represents the distance from flat earth perspective. On a round globe, this line would be curved. Secondly, the height of the detector is not measured exactly perpendicular to the surface of the Earth. However, such effects are expected to have minimal influence on the results, primarily due to the Earth's substantial radius relative to our distance L. For curious readers, measuring the length of 5km on a round earth would be $0,5$mm shorter than the completely flat ground, as if we were to *fly* the length L (flat line) compared to walking it (curved line).

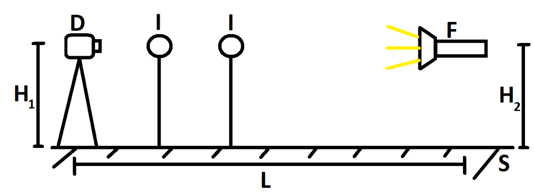

Figure 1: The experimental setup employs sea level as a reference point. Here, H_1 and H_2 denote the respective heights of detector D and flashlight F measured from sea level S. The iris diaphragms are designated as I, and L represents the distance between the detector and flashlight.

2.2 Cavendish's experiment

One experiment you could try is similar to the one Henry Cavendish conducted in 1797. He didn't set out to prove the shape of the Earth, but rather to demonstrate the existence of gravitational force as a property of objects with mass. This would be an even more fundamental result than playing with the shape of our planet.

In his work [1], Cavendish and his colleagues used two 158 kg objects to attract two lighter objects of mass 1.61 kg on the torsion pendulum. With this apparatus, he was able to calculate the gravitational constant G with just a 1% difference from the measurement obtained using modern equipment.

In the era of Cavendish, there was a widespread understanding in academic circles that massive objects, such as the Sun, Moon, and other celestial bodies, exert gravitational attraction force on each other. The only missing piece of the puzzle was the proportional constant. Allow us to present this concept to you through mathematical equations.

$$m\frac{d^2}{dt^2}x = F \tag{2}$$

We will start with Newton's motion equation (2). He established that the force F is directly proportional to the masses of both objects m and M, and inversely proportional to the square of the distance r ($F = G\frac{mM}{r^2}$). It may appear straightforward to divide both sides by m given its presence on both sides.

$$m\frac{d^2}{dt^2}x = G\frac{mM}{r^2}. \tag{3}$$

Certainly, here is the revised text:

The two instances of the variable m denote distinct properties. The m on the left side influences the body's deceleration due to inertia, while the other m represents the body's gravitational characteristics and its capacity to attract and be attracted by other objects with mass.

This experiment necessitates a high degree of precision, patience, and physical rigor to accurately evaluate the results. Even minimal movement within the vicinity can influence the experiment. To conduct this experiment, a torsion pendulum and two lightweight objects with mass m are required. Additionally, two significantly heavier objects with masses M are needed. Ideally, all four objects should have a round shape for the easiest determination of the centers of gravity of the objects. Position the light objects on the pendulum and place the heavy objects in close proximity so that the gravitational attraction between the objects induces a torsional motion in the same direction at both ends of the pendulum. Measure the distance r between the objects m and M on each side. The measured quantity in this instance is the acceleration $\frac{d^2}{dt^2}x$ of the light objects, which is to be determined using the pendulum. Subsequently, you can calculate your own gravitational constant G.

3 Our experiment

In the Middle Ages, sailors faced challenges with sea navigation until they discovered they could use the stars to guide their ships. In the northern hemisphere, there is a star named Polaris. While sailors couldn't pinpoint their exact location using the stars, they could determine the direction in which they were traveling.

Our experiment will be based on a similar concept. We will assume the rotation of the Earth around its axis and use this assumption to calculate the position of the stars. We will then measure the position of the stars to confirm or disprove our theory. Let's summarize the assumptions for this experiment:

- Earth is round.
- Earth rotates around its axis.
- Stars are much farther away than the local movement of the Earth around the Sun.
- The motion of the star (except Polaris) should be circular (counter-clockwise) around one point in the sky due to the (clockwise) rotation of the Earth.
- The position on the globe is given from outside sources.[1]

Let's discuss what we are trying to prove and what the consequences of our measurement will be, regardless of the results. We are proposing a model of the Earth as round and suggesting experiments to see if this model is plausible. It's important to note that if our results contradict our theory, it doesn't necessarily mean that the Earth is flat or has any other shape. Our assumption could simply be incorrect, there might be a systematic error in our measurement, or our analysis and interpretation could lead to the wrong conclusion.

In order to understand the stationary point in the sky during the Earth's rotation, it is important to consider why this phenomenon occurs. This fixed point is due to the Earth's relatively stable rotation axis over time. Observers at the North Pole would consistently observe the same celestial point when looking directly upwards, parallel to the rotation axis, regardless of the Earth's rotational speed. This principle also applies to observers at general latitudes, where despite local position changes during Earth's rotation, a consistent celestial point could be observed under specific directions. Such direction to the fixed point is represented by "s" and "s'" in Figure 1, while the straight-up direction is indicated by the gray lines.

Let us delve into our experimental procedure. The model of the Earth is depicted in Figure 3. Our initial step involves the identification of Polaris. This

[1] This may result in skepticism among conspiracy theorists due to the use of "corrupted" data. The inclusion of this point serves the purpose of demonstrating the proper setup of the apparatus. Alternatively, one could simply determine the direction using a compass or measure it from various angles.

can be achieved by obtaining the latitude α_{lat} from outer sources and determining the angle $\alpha = 90° - \alpha_{lat}$, subsequently orienting ourselves northward with the aid of a compass. In the event that the latitude and cardinal directions are unattainable or not pursued, an arbitrary angle, such as $\alpha = 45°$, may be selected. Precision in determining the angle α and direction is not imperative; the primary objective is to ensure the identification of Polaris and its subsequent recognition in the data.

Upon locating Polaris, we can proceed to consider the behavior of other stars. Let us illustrate the thought process from the North Pole once more. The concept in our model can be likened to gazing at the ceiling while spinning on an office chair. You may test this yourself. The outcome will consistently indicate that when you rotate by an angle α, the ceiling will rotate by an angle $-\alpha$ from its original position. Moving farther from the axis of rotation will not have an impact, provided we adjust the direction we are always facing to the fixed point. Using the known length of a day $\tau = 24\text{h}$, we can determine the rotational speed ω.

$$\omega = \frac{360°}{\tau} = \frac{15°}{h} \tag{4}$$

The rotational speed is 15° per hour. Consequently, conducting a qualitative verification of our theory should be feasible within one to two hours.

We are knowledgeable about the specific locations to observe, the optimal angles for positioning our equipment, and the appropriate duration for measurements. The most suitable recording device for this purpose is a cell phone, as the majority of models should have the capability to capture stars in the night. If this is not the case, adjusting the camera settings such as exposure length or ISO may yield better results.

When recreating our experiment, it is important to be mindful of the potential interference from ambient light, particularly from street lights. Additionally, it is advisable to use a mobile holder to ensure the stability of the camera's position. Regular and frequent photo documentation is essential when conducting such experiments.

The alternative approach involves the observation of the night sky from a consistent vantage point using a thin canvas to facilitate the recording of the positions of visible stars over an extended period. This technique, while seemingly straightforward, requires significant stability from the observer throughout the duration of the observation. Notably, a comparable method has been employed in historical contexts for the measurement of star positions 2.

4 Results (1.10.2023)

This section is dedicated to the analysis of the collected data. The data collection occurred on October 1st between 1:13 AM and 2:11 AM, utilizing a mobile holder to maintain the phone in a stable position at the appropriate angle. A

Figure 2: Drawing of ancient astronomer measuring the position of stars. Taken from [`www.nationalgeographic.co.uk`].

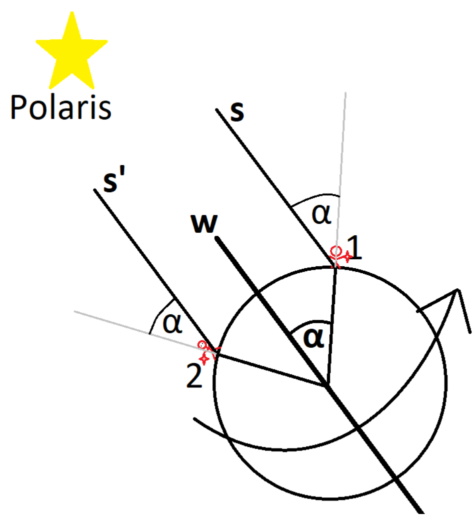

Figure 3: Our model represents the spherical shape of the Earth. The position of the observer is indicated by a red stick figure. The numbers 1 and 2 signify the same observer location at different times due to the Earth's rotation around its axis ω. The angle α serves as a parameter for calculating the observer's latitude, utilizing the formula $90^\circ - \alpha$. Lines s and s' correspond to the direction of Polaris, which is our fixed point. The gray lines illustrate the direction the red stick figure would be facing if looking directly upwards.

Figure 4: The image of the stars is highlighted by a red circle for enhanced visibility. We have also included only those stars that were consistently identified across all our photographs. These specific stars are underlined for clarity. The time at which each photograph was taken is indicated in the top right corner. A complete list of these photographs can be found in Appendix A.

photograph was taken every five minutes during this one-hour interval. Unfortunately, a significant number of the images proved to be unusable due to low contrast, which may have resulted from inadequate focusing or some form of automatic camera adjustment.

We identified several images that feature stars, enabling us to establish connections among them through comprehensive analysis. The positional data was acquired using software capable of interpreting the coordinates of each pixel; for instance, Microsoft Paint can serve this purpose effectively. This method of determining positions is both fast and exhibits minimal relative dispersion. In our analysis, we marked each visible star within the selected images, and one such instance is illustrated in Figure (4).

The movement of each star can now be tracked by analyzing their positions across individual images captured at various times. This analysis will yield a two-dimensional graph that illustrates stereographic projection. The relevant data is represented in the accompanying figure (5).

According to our model, we anticipate observing stars that exhibit circular rotational motion. In certain exceptional cases, the observed pattern may resemble the one depicted in Figure (6). It is evident that we are witnessing both circular motion and proportional displacements that vary according to the distance from the center. Given our limited understanding of the night sky, we can reasonably assume that the star represented in gray at the center is Polaris. Its movement also indicates that our measurement equipment may have

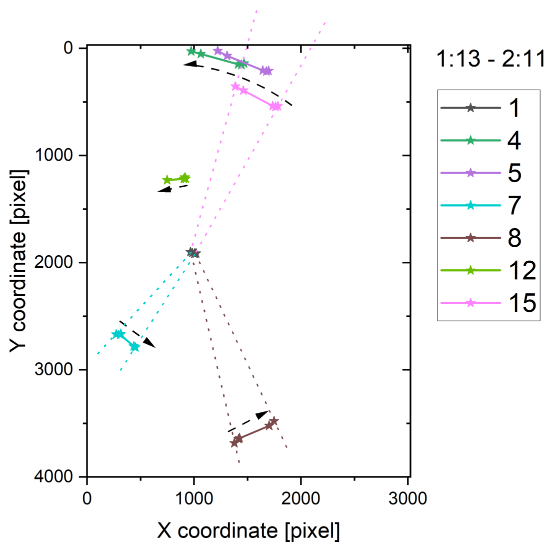

Figure 5: The graphical representation illustrates the position of various stars in the night sky. Dashed arrows denote the direction of movement over time, highlighting two significant properties. Firstly, these arrows form a circular pattern around the center of the diagram. Secondly, it is observed that stars located further from the center exhibit a more pronounced shift in their positions. Both characteristics align with our model of a spherical Earth.

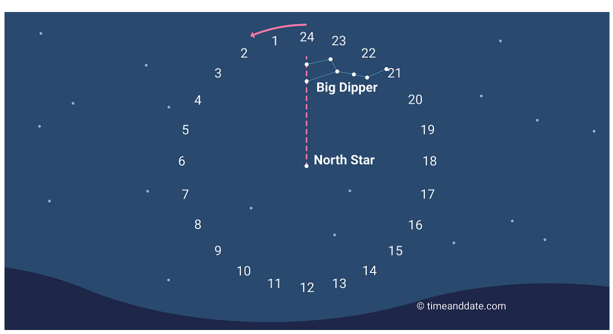

Figure 6: The ideal behavior of the night sky in the context of our model of a spherical Earth is largely unobservable due to the presence of daylight. However, during the period of polar night, the conditions may allow for observations that closely resemble these idealized predictions.

been subject to minor instabilities. Conversely, the angular displacements of the stars do not align with our initial calculations. We anticipated a constant angular shift slightly less than 15°; however, the measured values for the stars designated as 7, 8, and 15 were approximately 12°, 14°, and 16°, respectively. This discrepancy can be attributed to the limited amount of data collected, which resulted in significant statistical errors. With additional measurements, we should be able to obtain more consistent and accurate data.

5 Results (17.8.2024)

The methodology, camera equipment, and phone holder employed in this study are consistent with those utilized in Chapter 4, but with an increased frequency of photographic captures. The data presented in this chapter were collected on August 17, 2024, over a duration of one hour and six minutes, specifically during the time interval from 2:17 AM to 3:23 AM. This duration is adequate for observing an angular shift of 16.5°. The movement of each star is illustrated in the same format as in Chapter 4 in Figure 7. Notably, this measurement allowed for the capture and identification of a significantly greater number of stars across the majority of images. Furthermore, there was an observable improvement in stability compared to the previous measurement, yielding superior alignment between the measured and calculated angular shifts. The recorded shifts for all marked stars with dotted lines were determined to be $(16.5 \pm 0.3)°$.

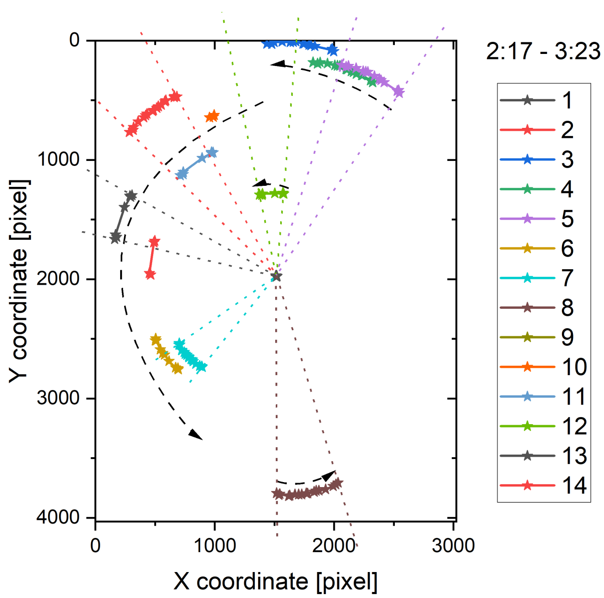

Figure 7: The graphical representation illustrates the position of various stars in the night sky. Dashed arrows denote the direction of movement over time. The data were taken on August 17, 2024, between times 2:17 AM and 3:23 AM.

6 Discussion

We created a round model of the Earth based on our assumption. Implications of such a model were mentioned and discussed as well as its intuitive understanding. It was a very limited and phenomenological model that did not explain the rotational movement of Earth nor the existence of a gravitation force as it was beyond the scope of this paper.

We correctly predicted the movement of stars and explained why we observed such things. Qualitatively it was successful in both instances of measurements. Both graphs 5 and 7 showed agreement with the rotational movement of stars and independence of angular shift on distance from the center, which was star Polaris.

The quantitative agreement was achieved solely during the second measurement conducted on August 17, 2024. The observed angular shifts were in precise alignment with the calculations provided by our model. The exceptional stability of our measuring equipment, combined with the high frequency of image acquisition, contributed significantly to this outcome.

The vast majority of the photographs were unusable, which significantly contributed to inaccuracies in the data analysis during the initial measurement. After reviewing the results, we decided to repeat the experiment. The unexpected drop in the quality of images during the shooting was compensated and better results were obtained.

7 Conclusion

In this paper, we presented several experimental methodologies dealing with roundness of the Earth. We also briefly discussed their underlying principle and the possibility of recreating those experiments. By employing certain reasonable assumptions, we successfully predicted the movement of celestial bodies and validated these predictions through experimental means. The results from our initial experiment exhibited small discrepancies; however, the anticipated trend was nonetheless identified. Conversely, the outcomes of the second experiment demonstrated strong congruence with our theoretical model, both qualitatively and quantitatively. The measured angular shifts were recorded as $(16.5 \pm 0.3)^\circ$, aligning closely with the theoretical value of 16.5°.

References

[1] A.S. Mackenzie, I. Newton, and H. Cavendish. *The Laws of Gravitation: Memoirs by Newton, Bouguer and Cavendish, Together with Abstracts of Other Important Memoirs*. Scientific memoirs. American book Company, 1900.

A Appendix

In this appendix, we will give you access to the data we measured and analyzed [`zenodo.org/records/14976259`]. The Zenodo repository consists of two folders named *01_10_2024* and *17_08_2024* which contain images with marked stars. Note to say, that